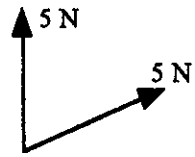
MECHANICS (A) UNIT 1 — TEST PAPER 5

Take $g = 9{\cdot}8\ \text{ms}^{-2}$ and give all answers correct to 3 significant figures where necessary.

1. Two forces, both of magnitude 5 N, act on a particle in the directions with bearings 000° and 070°, as shown.

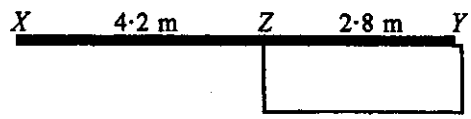


Calculate

(a) the magnitude of the resultant force on the particle, **(3 marks)**

(b) the bearing on which this resultant force acts. **(2 marks)**

2. A uniform plank XY has length 7 m and mass 2 kg. It is placed with the portion ZY in contact with a horizontal surface, where $ZY = 2{\cdot}8$ m. To prevent the plank from toppling, a stone is placed on the plank at Y.



(a) Find the smallest possible mass of the stone. **(4 marks)**

(b) State, with a reason, whether your answer to part (a) would be greater or smaller if a shorter portion of the plank were in contact with the surface. **(2 marks)**

3. A car, of mass 1800 kg, pulls a trailer of mass 350 kg along a straight horizontal road. When the car is accelerating at $0{\cdot}2\ \text{ms}^{-2}$, the resistances to the motion of the car and trailer have magnitudes 300 N and 100 N respectively. Find, at this time,

(a) the driving force produced by the engine of the car, **(3 marks)**

(b) the tension in the tow-bar between the car and the trailer. **(4 marks)**

4. A train starts from rest at a station S and accelerates at a constant rate for $2x$ seconds to a speed of $5x\ \text{ms}^{-1}$. It maintains this speed until 126 seconds after it left S and then decelerates at a constant rate until it comes to rest at another station T, $20x$ seconds after it left S.

(a) Sketch a velocity-time graph for this journey. **(4 marks)**

Given that the distance between S and T is 5·4 km,

(b) show that $x^2 + 7x = 120$. **(4 marks)**

(c) Find the value of x. **(3 marks)**

5. **i** and **j** are perpendicular unit vectors. The point A has position vector $6\mathbf{j}$ m relative to an origin O. At time $t = 0$ a particle P starts from O and moves with constant velocity $(5\mathbf{i} + 2\mathbf{j})$ ms^{-1}. At the same instant a particle Q starts from A and moves with constant velocity $4\mathbf{i}$ ms^{-1}.

(a) Write down the position vectors of P and of Q at time t seconds. **(3 marks)**

(b) Show that the distance d m between P and Q at time t seconds is such that

$$d^2 = 5t^2 - 24t + 36.$$ **(5 marks)**

(c) Find the value of t for which d^2 is a minimum. **(3 marks)**

(d) Hence find the minimum distance between P and Q, and state the position vector of each particle when they are closest together. **(4 marks)**

6. A, B and C are three small spheres of equal radii and masses $2m$, m and $5m$ respectively. They are placed in a straight line on a smooth horizontal surface. A is projected with speed 6 ms^{-1} towards B, which is at rest. When A hits B it exerts an impulse of magnitude $8m$ Ns on B.

(a) Find the speed with which B starts to move. **(2 marks)**

(b) Show that the speed of A after it collides with B is 2 ms^{-1}. **(3 marks)**

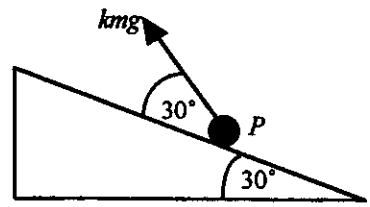
After travelling 3 m, B hits C, which is then travelling towards B at 2·2 ms^{-1}. C is brought to rest by this impact.

(c) Show that the direction of B's motion is reversed and find its new speed. **(3 marks)**

(d) Find how far B now travels before it collides with A again. **(6 marks)**

(e) State a modelling assumption that you have made about the spheres. **(1 mark)**

7. A particle P, of mass m, is in contact with a rough plane inclined at 30° to the horizontal as shown. A light string is attached to P and makes an angle of 30° with the plane. When the tension in this string has magnitude kmg, P is just on the point of moving up the plane.



(a) Show that μ, the coefficient of friction between P and the plane, is $\dfrac{k\sqrt{3}-1}{\sqrt{3}-k}$. **(7 marks)**

(b) Given further that $k = \dfrac{3\sqrt{3}}{7}$, deduce that $\mu = \dfrac{\sqrt{3}}{6}$. **(3 marks)**

The string is now removed.

(c) Determine whether P will move down the plane and, if it does, find its acceleration. **(5 marks)**

(d) Give a reason why the way in which P is shown in the diagram might not be consistent with the modelling assumptions that have been made. **(1 mark)**

